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ABSTRACT

One of the key challenges for the next-generation light sources such as X-FELs is to implement a timing stabilization and distribution system to enable ~ 10 fs synchronization of the different RF and laser sources distributed in such facilities with distances up to a few kilometers. These requirements appear to be beyond the capability of traditional RF distribution systems based on temperature-stabilized coaxial cables. A promising alternative is to use an optical transmission system: A train of pulses generated from a laser with low timing jitter is distributed over length-stabilized fiber links to remote locations. The repetition frequency of the pulse train and its higher harmonics contain the synchronization information. At the remote locations, RF signals are extracted simply by using a photodiode and a suitable bandpass filter to pick the desired harmonic of the laser repetition rate. Passively mode-locked Er-doped fiber lasers provide excellent long-term stability. The laser must have extremely low timing jitter, particularly at high frequencies (>1 kHz). Ultimately, the timing jitter is limited by quantum fluctuations in the number of photons making up the pulse and the incoherent photons added in the cavity due to spontaneous emission. The amplitude and phase noise of a home-built laser, generating 100-fs, 1-nJ pulses, was characterized. The measured phase noise (timing jitter) is sub-10 fs, from 1 kHz to Nyquist frequency. In addition to synchronization of accelerators, the ultra-low timing jitter pulse source can find applications in next-generation telecommunication systems.

Keywords: ultrafast optics, photonics, fiber, laser, RF, phase-noise, timing jitter

1. INTRODUCTION

One of the key challenges for the new fourth generation light sources such as the XFEL is to implement a timing stabilization and distribution system that allows the full exploitation of the potentially ~ 10 fs x-ray pulse for time-resolved studies. To this end, an ultra-stable timing and synchronization system must be implemented, covering the critical subsystems of the machine and the experimental area, which are spread over distances as large as several kilometers.

The electron beam needs to enter the undulator with timing jitter comparable to the pulse duration, which puts significant pressure on the synchronization system of the XFEL and requires point-to-point stabilization of various RF frequencies for the critical components (booster section, injector, bunch compressors and experimental area) with femtosecond precision. This translates to an amplitude and phase stability of the RF in the critical cavities of 10^{-4} and 0.01 deg respectively. The required amplitude stability levels have already been achieved in present day facilities, e.g. at JLAB and the DESY VUV-FEL.^{1,2} The best phase stability reported for superconducting cavities is on the order of 0.03 degrees; improving this to the 0.01 deg level (21 fs at 1.3 GHz) seems feasible. However, in order to accurately measure the phase stability, one requires a high-quality reference with much smaller phase jitter than the signal to be measured. A key challenge is to provide this reference in facilities spanning a few kilometers in length.

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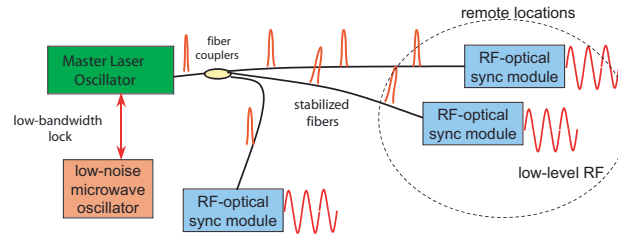


Figure 1. Schematic of the optical timing synchronization system.

These requirements on the timing stability appear to be beyond the capability of traditional RF distribution systems based on temperature-stabilized coaxial cables. A promising way to reach this goal is by using an optical transmission system,³ depicted schematically in Figure 1.⁴ A train of sub-picosecond pulses of light generated from a mode-locked laser with very low timing jitter is distributed over actively length-stabilized optical fiber links to an arbitrary number of remote locations. The precise repetition frequency of the pulse train, as well as its upper harmonics, contain the synchronization information. At the remote locations, low-level RF signals can be extracted simply by using a photodiode and a suitable bandpass filter to pick the desired harmonic of the laser repetition rate, or by phase locking an RF source to a harmonic of the pulse train.⁵

2. TIMING STABILITY REQUIREMENTS FOR THE X-RAY PULSES

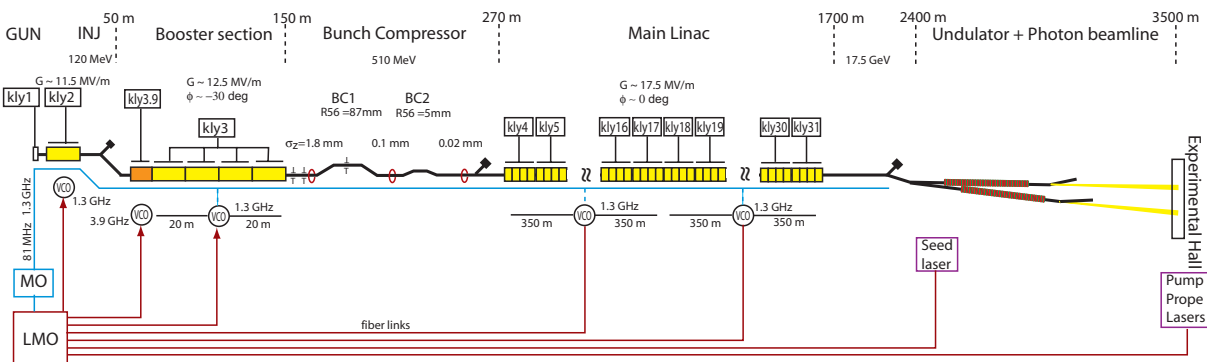


Figure 2. Layout of a 4th generation light source.

The layout of the European XFEL is shown in Figure 2 as an example of a fourth generation light source. The most critical sections of the machine in terms of inducing timing jitter onto the electron bunch are the injector and booster sections. Here, the electron bunch is accelerated off-crest to induce a chirp to make bunch compression possible. If amplitude and phase of the RF inside the cavities varies, this translates to a change in the centroid energy of the electron bunch, which is in turn directly converted into timing jitter by the bunch compressor chicane. A timing jitter on the order of the desired x-ray pulse width requires amplitude and phase stability of 10^{-4} and 0.01 deg respectively. This condition is relaxed by an order of magnitude in the main drive linac, as the on-crest acceleration makes the electron bunch less susceptible to amplitude and phase errors. If a seeding option is considered requiring an external laser system, as proposed for e.g. the FERMI facility,⁶ the timing jitter between the seed laser pulse and the electron bunch becomes a crucial issue for the quality of the x-ray pulse generated. Ideally, the seed laser pulse should have a flat-top profile and its pulse-width should be comparable to the electron bunch length. If a large timing jitter is present, the efficiency will drop dramatically and this timing jitter will directly transfer onto the final x-ray pulse. The seed laser is one of the components of the accelerator requiring the most stable reference and tightest lock possible. A similar argument is true for the probe laser systems in the experimental hall. To fully exploit the potential of a ~ 10

fs long x-ray pulse, the timing jitter between the probe pulse and the x-ray pulse must be kept to a minimum. Single-shot arrival time measurements might make it possible to sort the data taken afterwards according to the arrival time of the electron bunch, but a reliable arrival time measurement with 10 fs resolution has not been shown to date. Furthermore, the time it takes to accumulate a full data set increases drastically as a result of timing jitter between pump and probe pulses. Therefore, the probe laser systems also require a tight synchronization to the accelerator reference frequency, which makes a the stable distribution of that reference frequency to the experimental hall a key challenge.⁷ A possible solution is direct seeding of fiber amplifiers and Ti:Sapphire-based amplifiers with the optical pulses, distributing the timing information, or their second harmonics, respectively. This scenario has the inherent advantage that no timing jitter is added due to the generation of the seed pulse. Additional jitter imparted during the amplification process must be minimized.

3. MODE-LOCKED FIBER LASERS

Mode-locked fiber lasers are a natural choice to realize an optical master oscillator, because of the ease of coupling to the fiber distribution system, their excellent long-term stability, and the well-developed and mature component base available at the optical communications wavelength of 1550 nm. Recently, their technical capabilities have also improved significantly.^{8,9} Yb-doped and Er-doped fiber lasers offer stable and practical platforms for short pulse generation, at 1 μ m and 1.5 μ m, respectively. Fiber lasers can generate pulses from picosecond down to 35 fs in duration by simultaneous phase coherent lasing of multiple longitudinal modes spaced in frequency by the pulse repetition rate of the laser. During photodetection, these optical modes beat in the photodetector and generate all harmonics of the repetition rate within the bandwidth of the photodetector.

Mode-locking is initiated by a mechanism providing lower loss (hence, higher net gain) for a pulse than for cw radiation, leading to pulse formation from intra-cavity noise as soon as the laser is turned on. In the case of active mode-locking, this is a high-speed modulator. For passively mode-locked lasers, this is achieved by a real or artificial saturable absorber. For brevity, we restrict the following description to passive mode-locking. Once the pulses are shortened, the laser dynamics are dominated by an interplay of group velocity dispersion (different frequencies have different speeds) and Kerr nonlinearity (the refractive index depends on intensity), leading to the formation of soliton-like pulses, which intrinsically balance dispersion and nonlinearity.¹⁰ As the gain has a finite bandwidth, the generated pulses need to be stabilized by the saturable absorber, which favors the pulse and suppresses any cw-radiation. At the simplest level, short-pulse laser dynamics can be characterized by four processes: gain, saturable absorber, Kerr nonlinearity, and dispersion interacting in a periodic structure, defined by the physical cavity (Fig. 3a).

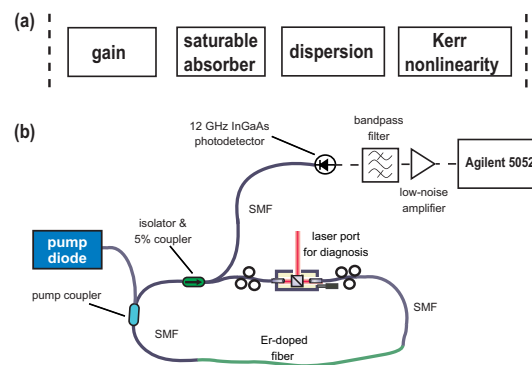


Figure 3. (a) The four effects governing spulse shaping in mode-locked lasers. (b) Schematic of the experimental setup: SMF, single-mode fiber.

In fiber lasers, the fiber assumes multiple roles, providing nonlinear and dispersive effects that dictate the soliton-like pulse shaping mechanism and moreover shielding against fast environmental fluctuations. The Er- or Yb-doped fiber segments form the gain medium, pumped conveniently by low-cost, fiber-coupled 980 nm diode lasers. A representative schematic of the laser is presented in Figure 3b, where saturable absorption is implemented by nonlinear polarization rotation in the fiber.

4. NOISE PERFORMANCE OF MODE-LOCKED FIBER LASERS

It is essential that the laser serving as the master oscillator has extremely low timing jitter, particularly at high frequencies (> 10 kHz), where further suppression through feedback is difficult. The timing of the pulse circulating in the laser cavity is affected by the intrinsic noise sources such as pump noise and amplified spontaneous emission noise from the amplification process. Ultimately, the timing jitter is limited by quantum fluctuations in the number of photons making up the pulse and the incoherent photons added in the cavity due to spontaneous emission.

The noise characteristics of mode-locked lasers is well-described using soliton-perturbation theory, along with quantum noise sources.^{11,12} These perturbations cause fluctuations in amplitude, phase, timing and center frequency. The last of these further contributes to timing in the presence of dispersion, *i.e.* a shift in center frequency is translated into timing shift *via* dispersion, which is known as the Gordon-Haus effect.¹³ For a fiber laser with small net dispersion and otherwise typical parameters, the quantum-limit is extremely small, in the order of 1 fs (from 1 kHz to 25 MHz, for a repetition rate of 50 MHz).

The noise performance of both an Er-doped fiber laser (EDFL) and an Yb-doped fiber laser (YDFL) were characterized. The EDFL is a stretched-pulse laser, implementing dispersion management¹⁴ producing pulses compressible down to 100 fs and 1 nJ of energy at a repetition rate of 40 MHz, centered at 1550 nm. A schematic of the EDFL and the experimental setup is shown in Figure 3. The YDFL is configured to produce pulses compressible down to 70 fs with 2 nJ energy content, with a repetition rate of 36 MHz, centered at 1030 nm.⁸ The YDFL (not shown) has a cavity similar to that of the EDFL. Both of the lasers are free-running, in the sense that the cavity length is uncontrolled and subject to slow, thermally induced fluctuations.

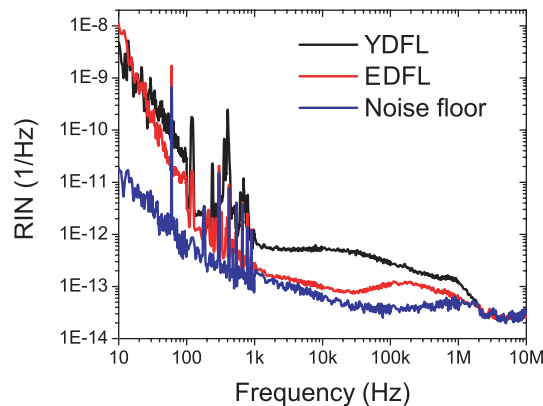


Figure 4. Relative intensity noise (RIN) of the EDFL and YDFL, along with the measurement noise floor.

The amplitude noise of the fiber lasers has been characterized. Figure 4 shows the relative intensity noise (RIN) of both lasers from 10 Hz to 1 MHz. The EDFL shows slightly lower high-frequency noise than the YDFL, which may be due to the different amplifier media and pulse shaping processes at work. The integrated RIN measured from 10 kHz to 1 MHz is about 0.04% rms for the YDFL, and 0.03% rms for the EDFL, compared to the average power level. Figure 5 shows the single sideband phase noise spectrum of the harmonic at 1.3 GHz extracted from the pulse train upon photodetection and filtering. This phase noise spectrum can be converted

into a timing jitter using $\Delta t = \frac{\sqrt{2 \int L(f') df'}}{2\pi f_0}$. The integrated timing jitter from 1 kHz to the respective Nyquist bandwidths, *i.e.*, half of the laser repetition rate, is measured to be about 18 fs and 10 fs for the YDFL and EDFL, respectively. For comparison, the phase noise of a very low noise frequency generator, a Marconi 2041, is also plotted. As the lasers are free running, the performance of the microwave oscillator is slightly superior in the low frequency regime (< 10 kHz), but at frequencies of ~ 100 kHz, the mode locked lasers reach a comparable level of stability, with the EDFL having the lowest noise among the three at frequencies higher than 20 kHz. Both lasers would be already suitable for an overall sub-100 fs timing distribution system, which is an important next step to achieve in several FEL facilities. On the long run, the EDFL seems to be a

stronger candidate for a master oscillator due to the availability of a larger variety of components at 1550 nm, and perhaps more importantly, transmission fibers with both signs of dispersion, which allow the construction of dispersion-compensated fiber links.

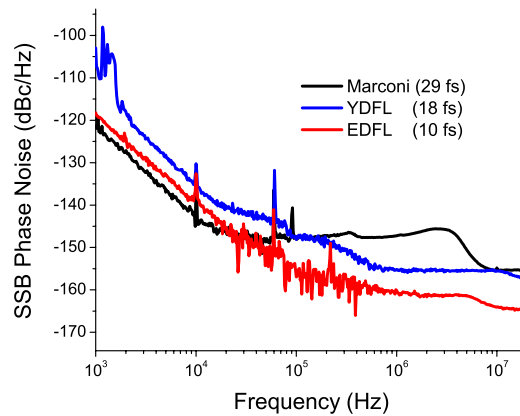


Figure 5. Single-sideband phase noise spectral density for the EDFL, YDFL, and a Marconi 2041 signal generator.

The measured timing jitter for both lasers is substantially higher than the noise limit given by the spontaneous emission noise. Several effects in the photodetection process lead to significant distortion of the actual phase noise spectrum. Due to the filtering of a single harmonic of the laser repetition rate, the power level of this harmonic is on the order of -20 dBm, even when saturating a typical photodetector. The thermal noise floor of the 50 Ohm resistor used to terminate the photodiode is at -178 dBm. That results in a minimum noise floor of -158 dBc for the single sideband phase noise. The phase noise measurement device employed in this study (Agilent 5052) implements a correlation technique, which provides an additional noise suppression of up to 10 dB. Even with this improvement, the phase noise of the reference oscillator itself constitutes another measurement limit. Another effect, which plagues phase noise measurements, is amplitude to phase conversion in the photodiode.¹⁵ The EDFL and YDFL have extremely low intensity fluctuations, nevertheless amplitude to phase conversion may contribute as much as 5 fs additional timing jitter.

5. MEASUREMENTS IN AN ACCELERATOR ENVIRONMENT

In order to verify that laboratory performance can be transferred to an accelerator environment without degradation, measurements were conducted at the MIT-Bates Linear Accelerator Center. We utilized a 500 m-long single-mode optical fiber link, which was already installed to achieve picosecond-stability optical signal transmission. The experiment consisted of three separate parts:

- (i) Locking of the EDFL to the S-band master oscillator at the Bates Facility to reduce the close-in noise of the laser system,
- (ii) Stabilizing the fiber link with a RF-based feedback to reduce the timing jitter added by the transmission to a few femtoseconds,
- (iii) Recovering a returning RF signal after 1 km of total travel through the fiber link with minimal added jitter.

The entire experiment was conducted over a time span of 3 weeks. The fiber laser worked reliably during this time without loss of mode-locking or significant increase of its phase noise.

5.1. LOCKING OF THE EDFL TO THE S-BAND REFERENCE

A schematic of the experiment is shown in Figure 6. The EDFL runs at a repetition rate of 40.22 MHz, such that the 72nd harmonic is at the desired synchronization frequency of 2.856 GHz. The laser is locked to the reference using a phase-locked loop (PLL), generating an error signal by comparing the 72nd harmonic of the

laser repetition rate to the S-band reference. This is fed back to a fiber stretcher, onto which 2 meters of the laser cavity fiber are wound. By controlling the fiber length, the repetition rate is adjusted. The unity gain point of the PLL has to be chosen carefully to ensure that the final phase noise of the signal is as low as possible. In fact, the EDFL becomes the *de facto* master oscillator of the facility. By locking to an RF reference for better low frequency stability, the resulting phase noise of the laser combines the good low-frequency properties of the RF oscillator and the excellent high-frequency properties of the free-running EDFL. Hence, a net improvement of the phase noise is achieved.

Figure 6. Schematic setup of the setup at MIT-Bates Laboratory.

Optical fibers exhibit a temperature dependent change of the refractive index, which causes arrival time jitter of a pulse propagating through the fiber. A typical value is $10^{-6}/\text{C}^\circ$,¹⁶ which corresponds to a fluctuation of 5 ps for a link length of 1 km and a temperature stability of 1 C° . This makes a stabilization scheme mandatory. Presently, there are two different approaches to stabilize the fiber length, both relying on a fiber stretcher with sufficient stretch to adjust for length fluctuations. One approach is to interferometrically stabilize the link: light from a single-frequency laser is sent through the link and beat with the returning, frequency-shifted light, thus generating an RF frequency equal to the difference frequency of the optical waves. The phase error of the optical waves is directly transferred into the recovered RF frequency. This results in a very high resolution phase detector and facilitates the locking.¹⁷ However, this scheme stabilizes the phase velocity and not the group velocity. If any information is transmitted over the fiber link, it will travel with the group velocity. The difference between these two velocities is on the order of 10^{-2} and any temperature and stress dependent changes are difficult to control.

The results for both open and closed loop scenarios stabilizing the MIT-Bates fiber link mentioned above, is shown in Figure7. If the loop is open, the inloop jitter in a bandwidth between 0.1 Hz and 5 kHz amounts

to 66 fs, which is reduced to 12 fs when the feedback is active. It should be emphasized that these results are by no means the limit achievable using this simple RF feedback approach. Using a higher harmonic determine the phase error in the mixer will lead to a linear increase in the resolution of the mixer. We expect to be able to decrease the residual timing jitter by 5-10 times if comparison frequencies in the 5-10 GHz range are used.

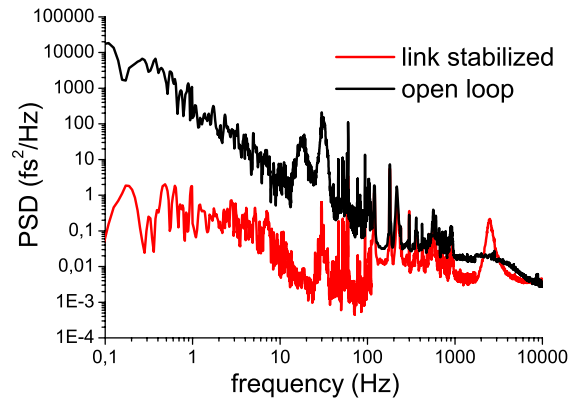


Figure 7. Mixer output signal of the RF fiber link stabilization.

We conclude that a fiber link of 500 meters length can readily be stabilized to the 10 fs level using RF techniques alone. For a fine lock, it is possible to employ optical cross-correlation of the pulse coming directly from the laser with the one returning through the fiber. Here, the resolution depends predominantly on the pulse width used. For the proposed distribution system with a pulse length of several hundred femtoseconds, the resolution of the cross correlator can easily surpass the resolution achieved by an RF-based feedback by a factor of 100 and is comparable to the resolution of the interferometric stabilization scheme. If optical cross correlation is used as a second, additional feedback scheme, a stability level of below a femtosecond is feasible.¹⁸ Here, both pulses are overlapped either inside a crystal for second-harmonic generation or on a two-photon absorption detector. This results in a strong signal with a duration on the order of the pulse length of the incident pulses. This increases the resolution of this optical phase detector by a factor of 50 compared to the RF approach.

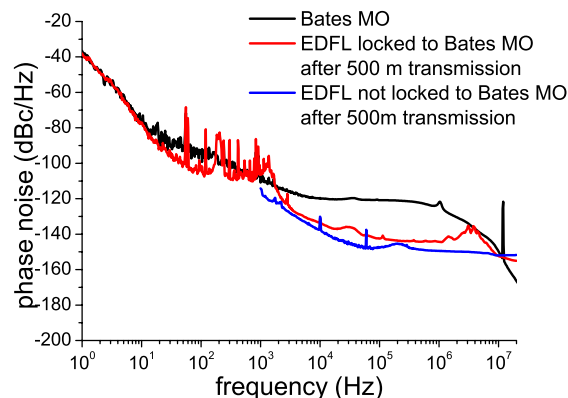


Figure 8. Single sideband phase noise of the Master Oscillator of the MIT-Bates Facility, the EDFL locked to the reference after transmission through the link and the free running EDFL.

5.3. RECOVERY OF THE RF SIGNAL AFTER TRANSMISSION

The crucial question for any timing distribution system is the transmission of various RF signals with the required level of stability. After stabilizing the fiber link (500 m-long) and locking the EDFL to a local RF source, the phase noise of the signal at the end of the fiber link is the relevant parameter. Figure 8 shows the phase noise of the RF master oscillator at the MIT-Bates Facility and of the EDFL locked to the RF source after returning through the fiber link (total travel distance of 1 km). The laser follows the RF master oscillator extremely well for the lower frequency range. For offset frequencies between 100 Hz and 10 kHz, there is technical noise added showing as spurs at 60 Hz and its harmonics. This is due to the driver of the pump diode and can be eliminated by running the pump diodes on battery power, or better isolation of the diode driver. At an offset frequency of 1.5 kHz, the free running spectra of the laser and the RF source meet, which corresponds to the optimal unity gain frequency for the PLL. As can be seen in Figure 8, the locked EDFL phase noise spectrum follows the free running laser spectrum closely (blue line). The difference in these two is due to technical issues in the photodetection process, rather than additional noise due to the fiber transmission. The phase locking of the EDFL to the MIT-Bates master oscillator source adds around 30 fs timing jitter. This is reduced to a few femtoseconds, if one ignores the technical noise spurs. The absolute phase noise is reduced, even taking the excess timing jitter due to the phase-lock into account, from 272 fs to 178 fs in a bandwidth of 10 Hz to 20 MHz. Including the residual noise of the fiber link stabilization, the complete system adds less than 50 fs of timing jitter in a bandwidth of 0.1 Hz to 20 MHz.

6. CONCLUSION AND OUTLOOK

In conclusion, mode-locked lasers producing sub-ps pulses can serve as ultra-low noise master oscillators for timing distribution in next-generation light sources. The main advantage of mode-locked fiber laser is the excellent noise performance at high frequencies, the high quality and availability of pump sources and components in the 1550 nm wavelength range. Measurements show a high-frequency performance surpassing that of microwave oscillators with ultra-low phase noise. We have demonstrated sources with record low timing jitter of 10 fs in a bandwidth of 1 kHz to the Nyquist frequency. Such sources can be made readily available for sub-100 fs and potentially sub-50 fs timing distribution. We have demonstrated the operation of a complete timing distribution system consisting of the master laser oscillator locked to the S-band reference oscillator and one 500 m-long stabilized fiber link. The residual inloop timing jitter due to the fiber link is 12 fs rms between 0.1 Hz and 5 kHz using an RF feedback system. Sub-fs timing jitter for the fiber link is feasible with optical cross correlation. The total added jitter due to locking of the laser to the microwave oscillator is 30 fs, leading to sub-50 fs of added jitter in a bandwidth of 0.1 Hz to 20 MHz. Overall, the way to a timing distribution system, with point-to-point jitter of 10 fs is outlined. Finally, it should be noted that the initial transition from a laboratory implementation of this scheme to operation in a real-world accelerator environment has been remarkably smooth.

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